VIII. VALUATION OF THE THREE MONTH CONTRACT AND ESTIMATION OF THE NPV AND CONVEXITY EFFECTS UNDER THE HULL-WHITE ONE FACTOR MODEL

48. In this section, I present my valuation of the Three Month Contract based on the Hull-White One-Factor model, which is an interest rate model that is commonly used in the industry and the same model used by Cont *et al.* (2011). I also discuss the methodology and findings presented by Cont *et al.* (2011).

A. Review of Valuation Framework

49. As discussed above in Section VII.D, the value of the Three Month Contract differs from the value of what appears to be an identical, non-cleared OTC interest rate swap. The difference in value arises from the daily cash flows associated with variation margin transfers under the Three Month Contract. Formally, the value of the Three Month Contract comprises two components: the value of an otherwise identical non-cleared OTC interest rate swap and the net present value of all daily variation margin transfers.

Value of Three Month Contract = *(Value of Non-Cleared OTC Swap)*
+
(Value of Movements in Variation Margin)

50. To value the Three Month Contract, one may value each of these two components separately. A value of a non-cleared OTC interest rate swap is determined by the swap's fixed rate; the schedule of fixed and floating payments; and the future term structure of interest rates, which indicates how the swap's floating payments are expected to evolve over time. The value of the swap is the net present value of cash flows based on these inputs.

51. The value of movements in variation margin depends on the term structure of interest rates and the volatility of interest rates during the life of the contract, as variation margin

flows when interest rates change. An interest rate model is used to estimate the volatility term structure and quantify how interest rates may evolve over time.

B. Valuation under the Hull-White One-Factor Model Applied to Swaptions

52. The Hull-White One-Factor model was first described by John C. Hull and Alan White in 1990.[48] Unlike many of its predecessor interest rate models, the Hull-White One-Factor model better fits real-world interest rates and interest rate volatilities, which is critical to pricing interest rate derivatives instruments correctly. To this day, the Hull-White One-Factor model remains a popular model for the evolution of future interest rates and is cited extensively in the academic literature, discussed in finance textbooks, and included in leading financial analysis software packages.[49]

1. Assumptions and Data Inputs

53. The Hull-White One-Factor model assumes that today's term structure of interest rates represents the long-term average of future interest rates. Certain economic events, such as changes in the rate of economic growth, may cause future interest rates to differ from this central tendency. When this happens, the model assumes that future interest rates will gradually return to their long-term average over time.

54. The Hull-White One-Factor model takes certain inputs that influence the predictions of the model; these inputs are the Hull-White One-Factor model's "parameters."[50] There

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[48] Hull, John, and White, Alan, "Pricing interest-rate derivative securities," *The Review of Financial Studies*, Vol. 3, No. 4 (1990) pp. 573–592.

[49] See Brigo, Damiano, and Mercurio, Fabio. *Interest rate models-theory and practice: with smile, inflation and credit*. Springer Science & Business Media, 2007, pp. 71-80; MATLAB Documentation, Financial Instruments Toolbox User's Guide, The Math Works, Inc., March 2015, Chapter 7, p. 6.

[50] The Hull-White one-factor model has two parameters. One of the parameters measures the volatility of short-term interest rates and the other parameter measures the speed with which interest rates return to their long-term average. This latter parameter is often referred to as the mean reversion speed parameter.

exist standard formulas based on the Hull-White One-Factor model that translate the model's parameters into prices of bonds, swaps, and other interest rate derivatives.

55. The specific values of the Hull-White One-Factor model parameters are commonly estimated, or "calibrated," to financial market data. Calibration consists of choosing parameter values such that the Hull-White One-Factor model (i) fits the current term structure of interest rates and (ii) predicts prices of interest rate derivatives that are consistent with the prices observed in the market.

2. Pricing and Calibration

56. In my analysis of the value of the Three Month Contract, I calibrate the Hull-White One-Factor model to publicly available data on short- and long-term interest rates and prices of interest rate swaptions.[51] I recalibrate the Hull-White One-Factor model on a daily basis during the Relevant Period. This ensures that my valuation of the Three Month Contract reflects then-current market conditions accurately.

57. I use the calibrated Hull-White One-Factor model to value the Three Month Contract under the valuation framework described above on each day during the Relevant Period. I define the fair coupon of the Three Month Contract as the fixed rate such that the value of the fixed cash flows equals the value of the floating cash flows. Further, I estimate the NPV and convexity effects for the Three Month Contract as the difference of the fair coupon rates and the Corresponding Rates. The sum of the NPV and convexity

[51] An interest rate swaption is a contract that grants the holder the right, but not the obligation, to enter into a certain interest rate swap at a certain time in the future. Hull (2011), p. 659. Swaption prices are commonly quoted for a wide range of option exercise terms and underlying swap maturities, each of which may range from one month to thirty years. I obtained the market data used in my analysis from Bloomberg, L.P.

effects reflects the amount by which the fixed payer benefits from variation margin under the Three Month Contract, relative to a non-cleared OTC interest rate swap.

3. Summary of Findings

58. **Exhibits 3A** to **3G** demonstrate my estimates of the NPV and convexity effects for the Hull-White One-Factor model calibrated to swaptions compared to the difference between DRW's bids and Corresponding Rates during the Relevant Period for several maturities of the Three Month Contract.[52] I find that my valuation of the Three Month Contract and daily estimates of the NPV and convexity effects are consistently in the range of DRW's bids during the Relevant Period across multiple maturities.

59. **Exhibits 4A** to **4G** demonstrate the same estimates of the NPV and convexity effects for the Hull-White One-Factor model calibrated to swaptions compared to the difference between the IDEX Curve and Corresponding Rates during the Relevant Period for several maturities of the Three Month Contract.[53] I find that my valuation of the Three Month Contract and daily estimates of the NPV and convexity effects are consistently in the range of the IDEX Curve during the Relevant Period across multiple maturities.

C. Valuation under the Hull-White Model Applied to Eurodollar Futures

60. As discussed above, the NPV and convexity effects are also present in other markets. As an alternative valuation approach, I use the NPV and convexity effects observed in the Eurodollar futures market to generate estimates for the parameters of the Hull-White One-Factor model.

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[52] Exhibits 3A to 3G also demonstrate my estimates of the NPV and convexity effects for the Hull-White One-Factor model calibrated to Eurodollar Futures compared to the difference between DRW's bids and Corresponding Rates, as described in Section VIII.C.

[53] Exhibits 4A to 4G also demonstrate my estimates of the NPV and convexity effects for the Hull-White One-Factor model calibrated to Eurodollar Futures compared to the difference between the IDEX Curve and the Corresponding Rates, as described in Section VIII.C.

61. The Hull-White One-Factor model provides a standard formula with which to calculate the size of this NPV and convexity effects that are embedded in the prices of Eurodollar futures contracts.[54] I employ this formula to estimate Hull-White One-Factor model parameters that are consistent with the implied par coupons, Corresponding Rates, and NPV and convexity effects shown in **Exhibits 2A** and **2B**.

1. Pricing and Calibration

62. Under this approach, I calibrate the Hull-White One-Factor model to publicly available data on prices of Eurodollar futures contracts. I recalibrate the Hull-White One-Factor model on a monthly basis to ensure that my valuation of the Three Month Contract reflects then-current market conditions.

2. Summary of Findings

63. **Exhibits 3A** to **3G** also demonstrate my estimates of the NPV and convexity effects for the Hull-White One-Factor model calibrated to Eurodollar futures compared to the difference between DRW's bids and Corresponding Rates during the Relevant Period for several maturities of the Three Month Contract. Similar to the Hull-White One-Factor model calibrated to swaptions above, I find that my valuation of the Three Month Contract and estimates of the NPV and convexity effects are consistently in the range of DRW's bids during the Relevant Period across multiple maturities.

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[54] See Hull, John C., *Options, futures and Other Derivatives, Eighth Edition,* Prentice Hall, 2011, p. 140 and Technical Note No. 1, "Convexity Adjustments to Eurodollar Futures," Available at: http://www-2.rotman.utoronto.ca/~hull/technicalnotes/TechnicalNote1.pdf.

The MATLAB software provides a function, liborfloat2fixed, that calculates the implied par fixed rate on a swap given prices of Eurodollar futures contracts. This function includes an option to calculate an adjustment for the NPV and convexity effects. This adjustment uses the Hull-White model adjustment formula discussed above. Using this MATLAB function, I choose Hull-White model parameter values such that the adjusted par fixed rates match the actual 7-year and 10-year Corresponding Rates during the Relevant Period as closely as possible.

64. **Exhibits 4A** to **4G** also demonstrate the same estimates of the NPV and convexity effects for the Hull-White One-Factor model calibrated to Eurodollar futures compared to the difference between the IDEX Curve and Corresponding Rates during the Relevant Period for several maturities of the Three Month Contract. I find that my valuation of the Three Month Contract and daily estimates of the NPV and convexity effects are consistently in the range of the IDEX Curve during the Relevant Period across multiple maturities.

D. Results from Valuation Analyses Corroborate the Results and Methodology of Cont *et al.* (2011)

65. I find that the methodology presented in Cont *et al.* (2011) is consistent with the valuation framework I have described above, which is based on an analytical model that is widely used and accepted in the field of financial valuation. However, the precise results of my valuation analyses are not identical to those of Cont at al. (2011). This is not at all surprising or concerning. Interest rate modeling involves the choice of modeling assumptions and data inputs. Differing specific assumptions and inputs naturally translate into differing valuations, as confirmed by the various analyses that I performed.

IX. DRW'S BIDS ON THE THREE MONTH CONTRACT WERE PLACED AT HIGHER RATES THAN THOSE ON A NON-CLEARED SWAP REFLECTING THE VALUE OF THE NPV AND CONVEXITY EFFECTS

66. In this section, I review DRW's bids made on the Three Month Contract. I explain that DRW was ready to, and indeed would likely want to, consummate any potential transaction on these bids on the Three Month Contract because:

a. DRW's bids were placed throughout the trading day and remained posted long enough to potentially attract a counterparty to trade during the Settlement Period;[55]

b. DRW's bids carried higher rates than those on a non-cleared OTC interest rate swap, thereby accounting for the presence of the NPV and convexity effects to reflect a value closer to fair market value, which would be more likely to attract a counterparty.

A. DRW's Bids Were Placed throughout the Trading Day and Remained Opened for Trading

67. **Exhibit 5** provides a summary of DRW's bids on the Three Month Contract. Based on the data that I have reviewed, DRW placed a total of 2,895 bids of which 1,165, i.e., 40% of bids, were placed during the fifteen-minute Settlement Period.

68. On average, the unmatched bids placed during the Settlement Period remained open for more than 17 minutes prior to deletion as shown in **Exhibit 6**. This time period is sufficiently long for potential investors to accept DRW's bids. In fact, the average 17 minutes between bid placement and deletion is an extremely long window of time, reiterating the fact that DRW was ready to trade at posted prices and provided ample opportunity for other investors to trade at these prices as well.

B. DRW Accounted for the NPV and Convexity Effects by Placing Bids at Rates Higher than the Corresponding Rates and Closer to All Legitimate Estimates of the Fair Value of the Three Month Contract

69. To adjust for the NPV and convexity effects on the Three Month Contract, DRW placed bids at rates higher than Corresponding Rates. **Exhibits 7A** to **7G** show graphs of

[55] The PM Settlement Period is between 1:45PM – 2:00PM Central Time. Complaint ¶28.

DRW's bids and the Corresponding Rates over the Relevant Period. **Exhibits 8A** to **8G** show graphs of the Corresponding Rates and IDEX Curve rates over the Relevant Period. For maturities of 7-, 10-, 12-, 15-, 20-, 25-, and 30-years, DRW's bids remained consistently higher than Corresponding Rates during the Relevant Period.

70. DRW's bids represented firm commitments to trade at the posted prices for notional values up to the posted quantities. As such, if DRW's bids had been hit by other traders in the market, DRW would have been committed to trade at these publicly-posted prices. IDCH affords no reneging on bids that are hit by other customers. Moreover, DRW's bids were competitive, in that they were posted at higher prices—and closer to all legitimate estimates of fair market values—than the Corresponding Rates. DRW's bids represent a proper use of the electronic exchange—to advertise a firm willingness to trade at better, more competitive prices to other traders.

X. WHETHER OR NOT A COUNTERPARTY HIT THEM, DRW'S BIDS WERE INVALUABLE FOR, AND A LEGITIMATE SOURCE OF PRICE DISCOVERY

71. In this section, I discuss the importance of price discovery in the context of the current matter. More specifically, I explain (i) what price discovery is in financial markets; (ii) the means by which price discovery is conducted by exchanges and clearinghouses; (iii) the nature of price discovery in illiquid markets; (iv) why regulators and market participants (*i.e.*, investors, exchanges, and clearinghouses) are strongly incentivized to participate in price discovery; and finally (v) the effects of price discovery on the settlement of futures contracts.

A. DRW's Bids Contributed to Price Discovery Because They Incorporated Relevant Information into the Market

72. In simple terms, price discovery, one of the central tenets of financial markets, is the process of determining the price of an asset, a good or a service. Finding the proper price involves analysis of demand or supply factors as well as other factors related to the transaction, the market, and available information.[56]

73. At its core, price discovery "involves the incorporation of new information into asset prices."[57] As O'Hara (2003) notes, "[t]raders with superior information will move prices toward full information levels … new information arrives, old information becomes stale, and even informed traders may face risks that their information is obsolete."[58] DRW's bids, at a premium rate that represented the NPV and convexity effects, certainly reflected new information in the marketplace relevant to the Three Month Contract price given that DRW had extensively studied the Three Month Contract[59] and its proper valuation, and modeled and incorporated the NPV and convexity effects into its bids.

74. The CFTC and its expert Mr. MacLaverty incorrectly conclude that an artificial price existed because IDCH established its IDEX Curve based on the Corresponding Rates before the Relevant Period, whereas during the Relevant Period it established its IDEX

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[56] Lehmann, Bruce N. "Some desiderata for the measurement of price discovery across markets." *Journal of Financial Markets* 5.3 (2002): 259-276 (hereinafter "Lehmann (2002)"). Lehmann (2002) similarly describes price discovery as "the efficient and timely incorporation of the information implicit in investor trading into market places."

[57] O'Hara, Maureen. "Presidential Address: Liquidity and Price Discovery," *Journal of Finance* 58.4 (2003): 1335-1354, p. 1339 (hereinafter "O'Hara (2003)").

[58] O'Hara (2003), p. 1351.

[59] Email exchange between Gerard Kopera and Garry O'Connor regarding "IDCG Curve Question from DRW," June 18, 2010, IDCG00002661-2 ("It seems that DRW is trying to understand our curve construction methodology with a degree of precision that we have not seen from other clients.")

Curve at rates that reflected DRW's bids.[60] Rather, a shift in pricing regimes across time is completely consistent with price discovery. A more appropriate view of the regime change in the IDEX Curve is that DRW brought new, valid information to the market, which IDCH incorporated to establish a new equilibrium curve.

75. DRW's contribution to price discovery is significant for several reasons. First, price discovery reduces uncertainty by providing market participants with a "fair and appropriate" price based on bids or offers on which market participants are genuinely willing to transact.[61] Reducing uncertainty decreases transaction costs, which in turn encourages trading and increases liquidity. Given the fact that no transactions for the Three Month Contract were consummated during the Relevant Period, DRW's bids were the primary source of price discovery in the Three Month Contract market. Second, the dissemination of information collected through price discovery also contributes to building investor confidence and maintaining stable financial systems.[62] As correspondence between DRW and IDCH indicate, IDCH directly encouraged DRW's bids during the Settlement Period.[63]

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[60] Complaint ¶3 and ¶34.

[61] IDCH Rulebook, p. 100.

[62] OECD, "New Approaches to Economic Challenges," April 2015. Available at: http://www.oecd.org/naec/NAEC_Projects-Overview_Reflection-and-Horizon-Scanning.pdf.

[63] "On December 15, 2010, IDCH informed DRW that it would not be necessary to change the settlement process if DRW could inject its quotes into IDEX's electronic system and by doing so such quotes would be reflected in the settlement prices. IDCH subsequently provided a list of third party vendors that could be utilized for injecting quotes. DRW entered into a licensing agreement with Sky Road, one such vendor and on January 24, 2011 began entering orders into IDEX's electronic system through the Sky Road system." DRW Letter to IDCG, Investigation Number IDCH II 2011-1, February 18, 2011.

Deposition of Gerard Kopera, April 2, 2015, pp. 73-74:
"**Q**. Do you recall any internal conversations at IDCH about DRW's intention to stream bids electronically? **A**. Yes.

B. Regulators Should and Do Promote Price Discovery

76. Financial regulators around the world, and particularly in the U.S., view price discovery as one of the most fundamental functions of financial markets. The SEC, the CFTC, and other regulators encourage and facilitate price discovery on financial markets through myriad rules and regulations predicated on improving price discovery. Moreover, the CFTC, in particular, relies on price discovery metrics to exert regulatory authority in relatively illiquid markets.[64]

77. The CFTC's commitment to price discovery is not new and has long served as an underpinning economic principle in the markets it regulates. The 1997 CFTC Annual Report notes: "Through effective oversight regulation, the CFTC enables the commodity futures markets better to serve their important functions in the nation's economy – providing a mechanism for price discovery and a means of offsetting price risk."[65]

78. The CFTC, throughout its rulemaking and policymaking processes, has emphasized its commitment to price discovery as a positive aspect of financial markets. In 1998, shortly before the Commodity Futures Modernization Act of 2000, the CFTC put price discovery front and center in its Statement of Regulatory Priorities: "The Commission's objectives are to: (1) Foster futures and option markets that accurately reflect the forces of supply or demand for the underlying commodity and are free of disruptive activity; (2) oversee

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Q. And what do you recall -- **A**. We were advocating for a while that we wanted customers to do that, so when clearing participants indicated that they were interested in actually participating in our marketplace, we were happy about that."

[64] See, for instance, CFTC (2009), "Significant Price Discovery Contracts on Exempt Commercial Markets; Final Rule" at http://www.cftc.gov/ucm/groups/public/@lrfederalregister/documents/file/e9-6044a.pdf

[65] CFTC 1997 Annual Report.

markets which can be used effectively by producers, processors, financial institutions, and other firms for the purposes of price discovery and risk shifting[.]"[66]

79. The CFTC also encourages robust mechanisms for price discovery, particularly for exchange procedures during settlement periods. The CFTC, through Core Principle L, dictates that a Designated Clearing Organization ("DCO") makes information about rules and operating procedures governing its clearing and settlement systems available to all market participants. The CFTC recognizes the importance of the settlement period for derivatives products, requiring regulated exchanges to publish and enforce rules that govern the settlement period, including the determination of settlement prices. The CFTC's Division of Swap Dealer and Intermediary Oversight (DSIO, formerly the Division of Clearing and Intermediary Oversight, or DCIO) evaluates the clearing and settlement procedures of all contracts. Included in this evaluation is a consideration of whether settlement prices and procedures are robust to potential manipulation or distortions. The settlement procedures for the Three Month Contract were vetted properly by the CFTC in this case.[67]

80. Core Principle L also requires a DCO to provide: (a) The terms and conditions of each contract, agreement, and transaction cleared and settled by the DCO; (b) the fees that the DCO charges its members and participants; (c) the DCO's margin-setting methodology, and the size and composition of its financial resources package; (d) daily settlement

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[66] CFTC, Statement of Regulatory Priorities, 63 FR 61402-01, November 9, 1998.

[67] See Deposition of Gerard Kopera, April 2, 2015, ¶¶19-20; Deposition of Michael K. Dundon, April 1, 2015, ¶114:10-12; Deposition of Robert Wasserman, April 22, 2015, ¶¶35-36.

prices, volume, and open interest for each contract the DCO settles or clears; and (e) any other relevant matter to clearing participants.[68]

81. As Chief Economist of the CFTC in 2008, I gave testimony to the U.S. House of Representatives Subcommittee on General Farm Commodities and Risk Management Committee on Agriculture regarding the importance of price discovery and the problems inherent in markets where price discovery is impeded. In a joint statement with John Fenton (then Director of Market Surveillance at the CFTC), I noted "Diminished hedging activity can also impair price discovery in futures markets since commercial hedgers typically are a primary source for new market information. Diminishing the ability of futures markets to serve their hedging and price discovery functions would likely have negative consequences for commerce in commodities and ultimately, for the nation's economy."[69] This same testimony included insight into the resources devoted to monitoring and improving price discovery in U.S. markets, noting "At the Commission, we are devoting, and will continue to devote, an extraordinary amount of resources to ensure that futures markets are responding to fundamentals and are serving the role of hedging and price discovery."[70]

82. Following the 2010 Dodd-Frank Wall Street Reform and Consumer Protection Act, the CFTC reiterated the importance of price discovery in markets that the CFTC regulates.

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[68] CFTC, Derivatives Clearing Organization General Provisions and Core Principles, Federal Register/Vol. 76, no. 216, CFTC 17 C.F.R Parts 1, 21, 39, and 140, RIN 3038-AC98, November 8, 2011.

[69] CFTC, Written Testimony of Jeffrey Harris, Chief Economist, and John Fenton, Director of Market Surveillance Before the Subcommittee on General Farm Commodities and Risk management, Committee on Agriculture, United States House of Representatives, May 15, 2008, p. 4.

[70] CFTC, Written Testimony of Jeffrey Harris, Chief Economist, and John Fenton, Director of Market Surveillance Before the Subcommittee on General Farm Commodities and Risk management, Committee on Agriculture, United States House of Representatives, May 15, 2008, p. 18.

The CFTC's Proposed Rules address "the specific requirements associated with protecting the price discovery function of trading on a DCM's centralized market as now specifically imposed by the Dodd-Frank Act."[71]

83. Similarly, price discovery serves to underpin the CFTC's Core Principles, which govern prospective regulation. Rules promulgated by the CFTC must be assessed under a cost-benefit analysis that lists price discovery as paramount to market and public concern. Section 15(a) of the Commodity Exchange Act requires the Commission to "consider the costs and benefits" of its actions within "five broad areas of market and public concern: (1) Protection of market participants and the public; (2) efficiency, competitiveness, and financial integrity of futures markets; (3) price discovery; (4) sound risk management practices; and (5) other public interest considerations."[72]

84. Futures markets that operate under the regulatory oversight of the CFTC as designated contract markets ("DCM," *i.e.*, board of trades or exchanges) are subject to certain rules and principles, one of which is to protect and facilitate price discovery. The CFTC's Core Principles state: "The board of trade shall provide a competitive, open, and efficient market and mechanism for executing transactions that protects the price discovery process of trading in the centralized market of the board of trade."[73] A CFTC representative has also reiterated a commitment to price discovery in this case, noting "The Division of Market Oversight I mean is a fairly descriptive term. That is to say they are looking at markets. They are trying to ensure that the markets serve their purposes

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[71] CFTC, Core Principles and Other Requirements for Designated Contract Markets, 75 FR 80572-01 Proposed Rules, CFTC 17 CFR Parts 1, 16, and 38, RIN 3038-AD09, December 22, 2010.

[72] CFTC, Core Principles and Other Requirements for Designated Contract Markets, 77 FR 36612-01 Rules and Regulations, CFTC 17 CFR Parts 1, 16, and 38, RIN 3038-AD09, June 19, 2012.

[73] CFTC, Code of Federal Regulations, §38.500 Core Principle 9, 17 C.F.R. §38.500, August 20, 2012.

of price discovery. There are no corners and such, so, essentially, they'd be looking at this from a market, a market integrity perspective."[74]

85. As such, price discovery is an important function of futures markets and a primary area of regulatory oversight for the CFTC. In my opinion, the CFTC's allegations in the current matter serve to dissuade market participants from contributing to price discovery because the CFTC is seeking to punish legitimate price discovery in a market where little price discovery existed prior to DRW's bidding activity.

C. Clearinghouses Also Promote Price Discovery and Recognize it as a Legitimate Economic Reason to Post Bids or Offers

86. Beyond regulatory requirements, clearinghouses have strong financial incentives to ensure that a market has a rigorous price discovery mechanism. As the counterparty to all traders, a clearinghouse needs to ensure that market participants are able to comply with all clearing and settlement rules, including those related to margining. While a clearinghouse is technically never long nor short a derivatives security, the clearinghouse suffers the loss if a participant defaults on a contract. Thus, the clearinghouse can diversify the risk of any one participant defaulting by attracting a greater number of clearing participants. Moreover, if settlement prices do not reflect true supply or demand, the margins collected by the clearinghouse may not adequately cover the risk of clearing member default. In this light, an effective price discovery market is also financially important and beneficial to the clearinghouse, as price discovery aids in ensuring that settlement prices reflect true supply and demand.

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[74] See Deposition of Robert Wasserman on Behalf of the CFTC, April 22, 2015, 40:1-7.